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Designing Reinforcement Layers of Railway Track Bed on the Basis of Theory and Practice

Andrzej Surowiecki
The International University of Logistics and Transport in Wroclaw

Piotr Saska
The International University of Logistics and Transport in Wroclaw

Abstract
The topic of the article is the design of reinforcement layers for the railway track bed on the basis of theory and practice. It focuses on presenting issues related to technical requirements for the track bed, requirements for track reinforcing layers, making strength calculations for the track structure, designing the thickness of the reinforcing layer and deciding on the right system. The article concludes with information on an innovative ground reinforcement material: the TriAxial geo-grids.

Keywords: rail track, reinforcing layers, subsoil

INTRODUCTION

Railway track bed reinforcement layers are nowadays used not only for new track construction, but also for the modernisation and renovation of railway lines. They are used to increase the load-bearing capacity of the subsoil in view of the increasing trend to adapt major railway lines for increased traffic and higher speeds. The reinforcing layers may be individual inserts (e.g., geosynthetic mats) or a conglomerate (a layer of aggregate spread on a geosynthetic grid). Reinforcing layers are selected taking into account a number of parameters of the railway track and the reinforcing system.

Therefore:
- the technical requirements for the track bed should be reviewed,
- the requirements for the track reinforcement layers should be analysed,
- strength calculations of the track structure should be made [24],
- the thickness of the reinforcing layer should be designed and a decision should be made on the selection of the appropriate system.

These issues are presented and discussed in this paper (article).

1. TECHNICAL REQUIREMENTS FOR THE TRACK BED

The upper zone of the track bed (known as the subgrade) should be designed with an assumption of its sustainability of 20÷50 years, depending on the operating parameters of the railway line. The upper part of the subgrade should be characterised by appropriate [11]:
- ad hoc strength (load-bearing capacity and stiffness),
- in-service strength (durability),
- homogeneity.

The track bed (subgrade floor) should have a level of bearing capacity and stiffness that guarantees during the operation process:
- the occurrence of tensions not exceeding the permissible value,
- a minimum value of the modulus of deformation (Table 1).

Tab. 1. Minimum values of the modulus of deformation of the subgrade, measured on the trackway *E*, MPa [11]

Speed v_{max} [km/h]	Intensity of transport T (Ti/year)			
	T ≥ 25	10 ≤ T < 25	3 ≤ T < 10	T< 3
1	2	3	4	5
$200 < v_{max} \leq 250$	120 (80)	120 (80)	120 (80)	110 (70)
$160 < v_{max} \leq 200$	120 (80)	120 (70)	110 (60)	100 (55)
$120 < v_{max} \leq 160$	120 (70)	110 (60)	100 (50)	90 (45)
$80 < v_{max} \leq 120$	110 (60)	100 (55)	90 (45)	80 (40)
$v_{max} \leq 80$	100 (50)	90 (45)	80 (40)	80 (40)

The modulus values in front of the brackets in this table are the required values for the new-build and upgraded track bed for speeds v_{max} > 160 km/h. The modulus values in parentheses are the required values for the track bed of lines in service – they should be used when assessing the need to reinforce the track bed and when designing its repairs.

The durability of the upper part of the subgrade should be ensured by the incorporation of soil material or other materials [11]:
1. sufficiently stable, i.e:

   - containing no soluble substances, e.g., salts;
   - containing no more than 0.2% organic matter;
   - with a sulphate content of not more than 0.2%,
2. well-graded, i.e. well compacted and not affected by vibration,
3. non-swelling (frost resistant),
4. mechanically stable at the interfaces of the individual layers, i.e., not mixed with other ground materials; this condition must be fulfilled in particular at the interface with the sub-ballast bed in which the sleepers are embedded (the sub-ballast bed is made of crushed stone),
5. adequately water-permeable; the water permeability coefficient k_{10} for the soil immediately beneath the ballast should be:
   - $k_{10} \geq 1 \cdot 10^{-4}$ m/s, where the function of the ground cover is to allow rainwater to pass through; e.g., a filter layer on a station level;
   - $k_{10} < 1 \cdot 10^{-6}$ m/s, when it is necessary to prevent infiltration of rainwater into the subgrade (the track should then be sufficiently hardened and profiled with a transverse slope towards the drainage),
6. preventing migration of fine particles from the substructure towards the ballast; this requirement is met by aggregates containing 10÷20% of grains smaller than 0.2 mm.

2. REQUIREMENTS FOR TRACK REINFORCEMENT LAYERS

Track bed reinforcement layers (often referred to as protective coverings) are an intermediate layer between the track bed ballast (made of stone ballast) and the floor of the track subsoil (subgrade). The protective coverings are used [11]:

- as a safeguard where the technical requirements for the upper parts of the substructure are not fulfilled,
- when the requirements for the upper layers of the substructure are fulfilled, but the covering will cause an improvement in another element of the substructure, e.g., preventing the inflow of rainwater to the deeper cohesive soil layers in the track bed.

Protective covering structures (also called protective layers), are designed on railroad lines depending on local water, soil and operational conditions. Therefore, protective coverings can be used, among others, to:

- enlarge the bearing capacity of the subgrade, modify the distribution of forces and change the stiffness of the soil in the subgrade,
- improve the dynamic parameters of the subgrade,

- improve the filtration conditions at the interface between the subgrade and the ballast,
- protect the subgrade soils from water, erosion or frost,
- drain rainwater.

The thicknesses of typical track protection layers for the subgrade of a new-build or extension are adjusted to the subgrade soil class as follows [6]:
- for class QS1 → minimum thickness of protective layer $g_{o\ min}$ = 0,50 m,
- for class QS2 → $g_{o\ min}$ = 0,35 m,
- for class QS3 → $g_{o\ min}$ = 0,30 m.

Technical Conditions Id-3 [11] gives the following classification of soil quality for subgrade construction or repair:
- class QS0, which includes, among others: organic soils; susceptible soils (low-strength soils, containing more than 15% of fine particles, soggy or unconsolidable); thixotropic soils; contaminated soils (e.g., industrial waste); plastic soils containing more than 15% of fine particles, collapsible or swelling soils,
- class QS1: rocks that are very or moderately susceptible to weathering; soft rocks (e.g., rocks with the Los Angeles LA factor > 40),
- class QS2: soils containing 5-15% fine particles, except for sunken soils; homogeneous-grained soils (index U < 6) containing less than 5% fine particles, except for sunken soils; moderately hard rocks (e.g., rocks with the Los Angeles factor $30 > LA \leq 40$),
- class QS3: well-graded soils (index U ≥ 6) containing less than 5% of fine particles; hard rocks with the Los Angeles coefficient $LA \leq 30$.

Protective coverings can be single or multilayer. Various materials are used to reinforce and protect track beds [17]:
- in loose form (e.g., soil materials, aggregates),
- in liquid form (e.g., asphalt emulsions),
- in the form of masses (e.g., stabilized soils, asphalt concrete),
- in the form of rolls, sheets and plates (e.g., geosynthetics, reinforced concrete slabs).

As a rule, they are made of mineral soils, such as unsorted fine aggregate, gravel, sand and aggregates (unsorted stone, wedge, grit), taking into account the following principles [11]:
1. where it is necessary to reduce the thickness of the ground protection layer to be incorporated, stabilisation of this soil with binders, e.g., cement, lime, bitumen, or resins, may be used,

2. thin permeable (e.g., geotextiles, geonets) and impermeable (e.g., foils, bituminous coatings) coverings are used as reinforcement and multilayer protection to reduce the thickness of the necessary track bed and to meet the specified requirements,
3. waste materials may be used only if all requirements for materials suitable for the construction of track bed protective layers are met.

Currently, on the upgraded Polish State Railways lines, track protection coverings are usually built in the form of layers of suitably selected soil or aggregate, if necessary, placing geosynthetic mats such as separation or filtration non-woven geofabric and reinforcing geonets underneath.

When using geotextiles and geofabrics, the following principles shall be taken into account [11]:
- geotextiles and geotextile separation fabrics should be laid under protective layers where the material of these layers does not meet the requirements for contact with the subgrade soil;
- in the case of soggy crossings, it is recommended to spread a separation and filtration geotextile over the entire width of the track bed (Fig. 1),
- when dimensioning track bed reinforcement, taking into account the reinforcing effect of geotextiles and geofabrics is not recommended,
- the installed geotextiles and geofabrics should be as incompressible as possible, as their high compressibility may hinder the process of compaction of the protective layer and achievement of sufficient load-bearing capacity of the track bed immediately after completion of the works.

Reinforcing geonets (which are layers of reinforcement) should be used taking into account the requirements of [11]:
1. reinforcement of the protective layer with a geogrid is performed when:
   - the thickness of the necessary protective layer exceeds 0,40÷0,45 m,
   - it is necessary to reduce the total thickness of the substructure, e.g., due to local water and ground conditions,
   - it is advisable to build a protective layer of uniform thickness over a longer section.
2. in principle, the geogrid should be laid only in the service load zone, i.e., at a width of 3,80÷4,20 m (Fig. 1),
3. in case of expected settlements of the track bed, convex bends should not be used,

4. dimensions of geogrid meshes should be selected to ensure wedging of aggregate grains in them; for this reason, it is advisable to lay the geogrid in the aggregate layer at the level of 0.05÷0.10 m above the bottom of the layer,
5. the minimum thickness of the aggregate layer placed on the geogrid should be 0.20 m (if the calculated thickness is less, it should be increased) and no more than 0.30 m in the case of layers made of gravel and unsorted fine aggregate bed and no more than 0.50 m – if the layers are made of crushed aggregate (if the thickness of the substructure needed is greater, then another geogrid and another layer of aggregate should be placed),
6. overlaps when joining geogrids should not be smaller than 0.50÷0.80 m in the case of weak subsoil and 0.40 m – when the subsoil has adequate strength.

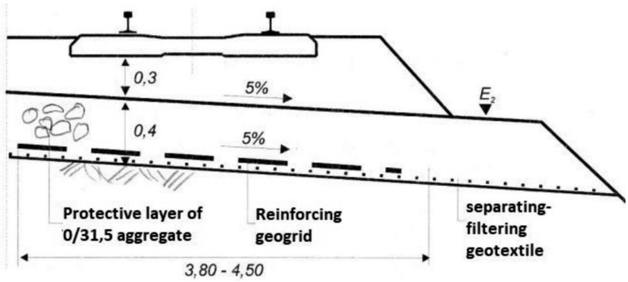

Fig. 1. Example of track reinforcement with geotextile materials [11]. Layering arrangement from below: separation and filtration geotextile, reinforcing geogrid, protective layer of 0/31.5 aggregate

3. AN OUTLINE OF STRENGTH CALCULATION AND DIMENSIONING OF THE TRACK STRUCTURE

When designing reinforcing layers for the railway road bed, it is essential to know the relevant strength and dimensioning calculations for the track superstructure. These calculations are accompanied by the adoption of a track model. Usually, the track model is a beam of infinite length located on a continuous and elastic foundation (Fig. 2). This model makes it possible to calculate the elastic deflections of the pavement due to static loading [24, 25].

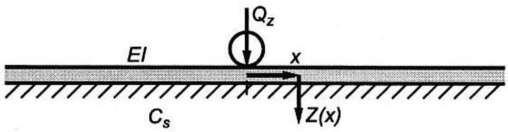

Fig. 2. Model of the track as a beam mounted on a continuous elastic base [24, 25]: Q_z – vehicle wheel force [kN], C – substrate elasticity coefficient [kN/mm^3], EI – bending stiffness of the track rail [kNm^2], C_s – track elasticity coefficient [kN/mm]

After assuming a linear relationship between the unit ground pressure q and the corresponding deflection $z\,(x)$, the following is obtained:

$$q = C \cdot z\,(x)\ \ [\mathrm{kN/m^2}] \quad \text{or} \quad q = U \cdot z\,(x)\ \ [\mathrm{kN/m^2}] \tag{1}$$

where:

C – substrate coefficient for a ‘weak’ substrate C = 2 MN/m^3, for a ‘load-bearing’ substrate C = 20 MN/m^3, U – rail base elasticity module which determined the volume of even loading per a length unit of the rail base, thereby causing its individual bend; for a ‘weak’ substrate U = 9 MPa, for a ‘load-bearing’ substrate U = 90 MPa.

The substrate coefficient C is correlated with the modulus U through a formula of the form:

$$U = C\,\delta_p\,a_p\,b_p\,(2\,l_p)^{-1}\ \ [\mathrm{MPa}] \tag{2}$$

where:

δ_p – sleeper deflection factor,
a_p, b_p – respectively the length and width of the sleeper,
l_p – sleeper centre distance.

The track elasticity coefficient C_s can be expressed by the bending stiffness of the rail EI [kN m^2]:

$$C_s = 8\,k^4\,l_p\,EI\ \ [\mathrm{N/mm}] \tag{3}$$

where k is the parameter expressed by the formula $k = [U\,(4\,EI)^{-1}]^{0{,}25}$.

For a ‘weak’ substrate C_s = 5,5 kN/mm, for a ‘load-bearing’ substrate C_s = 55 kN/mm.

The deflection of the rail is expressed by a fourth-order differential equation:

$$EI\ d^4z/dx^4 + U\ z = 0 \tag{4}$$

When the parameter k^{-1} is taken as $L = [4\ EI\ (U)^{-1}]^{0,25}$ [m], the relationship between the length of the track deflection (the so-called equivalent length L of the track, within $L = 0,7 \div 1,3$ m) and the elastic modulus of the rail substrate U is obtained. Then, equation (4) becomes:

$$d^4z/dx^4 + 4\ z\ (L)^{-1} = 0 \tag{5}$$

The solution to this equation is the deflection of the rail $z\ (x)$:

$$z = Q_z\ (2\ UL)^{-1}\ \eta\ x\ (L)^{-1} \tag{6}$$

From the above equation, the bending moment M has the value defined by the relation:

$$M = 0,25\ Q_z\ L\ \mu\ x\ (L)^{-1} \tag{7}$$

in which:

μ is the influence line of the bending moment for the vertical vehicle wheel load $Q_z = 1$.

Knowing the deflection of the rail $z\ (x)$, the rail pressure on the sleeper P is calculated:

$$P = Q_z\ l_p\ (2\ L)^{-1}\ \eta\ x\ (L)^{-1} \tag{8}$$

where:

l_p - length of sleeper,
η – deflection influence line for vertical vehicle wheel load $Q_z = 1$.

Another issue in the strength calculation of the railway superstructure is the estimation of the tensions: in the rail, in the sleeper and in the ballast. Usually, these calculations are made in an approximate manner, assisted by the scheme shown in Figure 3 [24, 25]. The force from the vehicle acting on the railhead is decomposed into vertical Q_z and horizontal Y components. These forces work eccentrically on the rail. The Y force is converted into an identical Y force acting on the rail foot and a concentrated moment Ms applied at the same point as the Y force. Fig. 3 also illustrates the distribution of normal tensions in the rail foot from: the moment M_s, the horizontal force Y and the vertical force Q_z and a summary diagram of these tensions [24, 25].

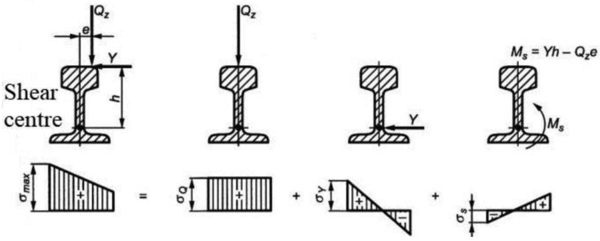

Fig. 3. Nature of rail loading and tension diagrams acting in the foot of the rail [24, 25]

The maximum bending tension is [24, 25]:

$$\sigma_{max} = k_d \, \sigma^* \tag{9}$$

where:

k_d - dynamic factor,

σ^* - average tension, calculated as follows:

$$\sigma^* = Q_z L \, (5 \, W_s)^{-1} \tag{10}$$

In this formula, W_s is the strength modulus in relation to the rail foot.

The value of the permissible tension in the rail taking into account the fatigue strength of the rail steel is calculated from the formula:

$$\sigma_{dop} = R_e \, (1{,}3)^{-1} \tag{11}$$

where:

$R_e \approx 0{,}64 \, R_m$ is the yield strength of the rail steel, R_m – resistance to tearing.

The average value of the tension occurring in the head of the rail $\sigma^{gs}{}_{average}$ in the wheel-rail contact area (called contact tension) is calculated using the following formula [24, 25]:

$$\sigma^{gs}{}_{average} = Q_z \, (r \, b)^{-1} \, \{\pi \, E \, [64 \, (1 - \upsilon^2)]^{-1}\}^{0,5} \quad [\mathrm{MPa}] \tag{12}$$

in which:

Q_z – vertical force [kN] from a vehicle wheel with radius r [mm].

b – half the width of the wheel-rail contact area,

E – elasticity modulus,
υ – Poisson's ratio.

For the $E = 2{,}1 \cdot 10^5$ MPa module, Poisson's ratio $\upsilon = 0{,}3$ and $b = 6$ mm, we obtain:

$$\sigma^{gs}_{average} = 1374\ [Q_z\ (r)^{-1}]^{0,5} \quad [\text{MPa}] \tag{13}$$

Vertical tension $\sigma^{s\text{-}p}_{vertical}$ at the level of the rail foot, transferred from the pad to the sleeper, is calculated in accordance with the formula [24, 25]:

$$\sigma^{s\text{-}p}_{vertical} = \beta\ P_{average} \tag{14}$$

where:

β – dynamic factor

$P_{average}$ – the average value of the force transmitted by the rail to the sleeper, calculated from the formula:

$$P_{average} = Q_z\ l_p\ (2L)^{-1} = 0{,}5\ Q_z\ [k_d\ (l_p)^3\ (4EI)^{-1}]^{0,25} \quad [\text{kN}] \tag{15}$$

in which:

Q_z – vertical force transmitted by the vehicle wheel [kN],
l_p – axial distance of the sleepers [mm],
L – equivalent length of the track [m],
EI – bending stiffness of the rail [kNm^2].

The replacement length of the track is calculated from the following formula [24, 25]:

$$L = [4\ EI\ (U)^{-1}]^{0,25} = [4\ EI\ l_p\ (C_s)^{-1}]^{0,25} \tag{16}$$

where:

C_s – the stiffness of the track support in the contact zone of the sleeper and ballast [kN/mm].

The permissible tensions at the rail-substrate contact are assumed as follows:

- for wooden sleepers made of softwood 1.00÷1.50 MPa,
- for wooden sleepers made of hardwood 1,50÷2,50 MPa,
- for concrete sleepers 4.0 MPa.

The tensions transmitted by the sleeper to the ballast are expressed by the formula:

$$\sigma_{p\text{-}p} = Q_{max}\,[(a_{pd} - s_s)\,b_{pd}]^{-1}\ \ [\mathrm{MPa}] \tag{17}$$

in which:

a_{pd} – length of the sleeper [mm],
b_{pd} – width of the sleeper [mm],
s_s – axial distance of the rails [mm],
Q_{max} – maximum force transmitted by the vehicle wheel [kN].

The maximum permissible tension at the contact surface between the sleeper and the ballast is taken as 0.50 MPa. However, the vertical tension in the ballast at the depth *"z"* under the sleeper is calculated as follows [24, 25]:

$$\sigma_z = 1{,}50\ Q_{max}\ \{[3\,(a_{pd} - s_s) + b_{pd}]\ z\ \mathrm{tg}\ \eta\}^{-1} \tag{18}$$

where:

$\eta = (30\text{-}36^{o})$ is the angle of tension distribution in the ballast.

4. DESIGN OF TRACK BED REINFORCEMENT LAYERS

4.1 Design principles [17, 24]

The design of protective coverings is carried out with appropriate calculation methods and by adopting simplified recommendations, developed on the basis of operational observations. For example: the BR railways dimension reinforcement systems on the basis of tensions in the track, while the DB railways dimension reinforcement systems on the basis of strain modules (Fig. 4) [17, 24].

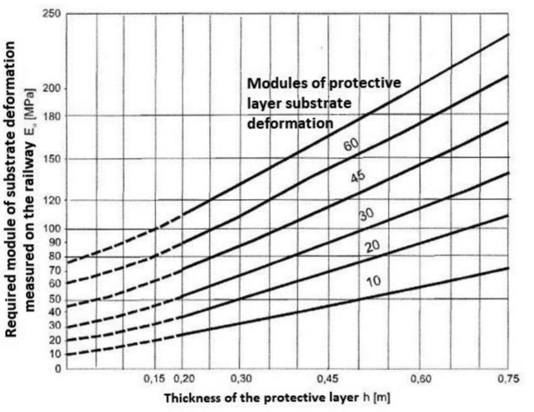

Fig. 4. Nomogram for the dimensioning of protective layers in trackways used on the DB railways (based on the modulus of elasticity) [17, 24]

The varying thicknesses of recommended reinforcing layers on different EU railways (Fig. 5) are due to different approaches to the problem. An attempt to unify the dimensioning methods is the work of the D117 ORE Appraisers Committee, whose results are shown in Fig. 6 [17]. Based on these studies, in the UIC 719R Charter, a unified method was recommended, which takes into account [4, 17]:

- vertical axle load of the rail vehicle,
- train speed,
- loading of the railway line,
- type of subsoil beneath the track.

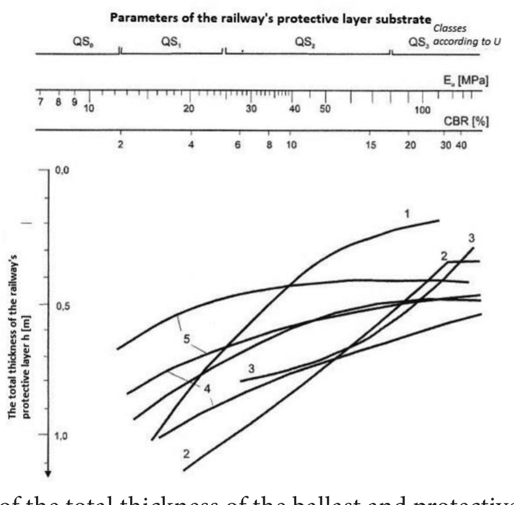

Fig. 5. Comparison of the total thickness of the ballast and protective layers of the track bed [17]: 1 – track with a loading of 0,10 Tg/day (on the basis of the CBR indicator);

2 – main lines of DB (for the trackbed deformation modulus E_o = 120 MPa); 3 – main lines of SBB (on the basis of the deformation modulus) 4 – lines of SNCF adapted to high speed (on the basis of the soil class); 5 – lines of the categories 1 and 2 of UIC in accordance with the D117 ORE Appraisers Committee

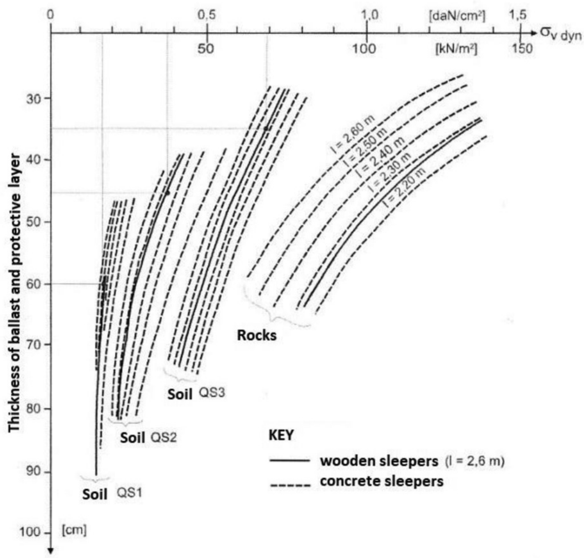

Fig. 6. Required total thickness of ballast and track bed on the UIC Class 1 and 2 lines depending on the subgrade soil classes [17]

In the handbook [17], it is shown that the two dimensioning methods (the tension method and the method based on strain moduli) are not comparable, because:

- impacts of the VSS plate for test loads are limited to a depth of approx. 1.0 m,
- the tension method takes into account the different operational influences occurring on the individual railway lines and the highest tension quotients resulting from the analysis of multiple layers of the substructure, including layers at considerable depths.

For this reason, the *Technical Conditions* [11] recommend practising both methods and taking the average thicknesses of the reinforcing layers, but not less than the thicknesses resulting from calculations based on the modules of deformation. In the dimensioning process, it is recommended to take into account [17]:

- stiffness of the substructure (required design modulus of elasticity of the substructure measured in the track plane),

- the permissible tensions for the soils of the upper layers of the substructure,
- mechanical stability of the subgrade soils at the interfaces of the individual layers,
- resistance of soils in the upper layers of the subgrade to vibrations generated by vehicle traffic,
- frost resistance of soils in the upper layers of the subgrade.

The design and dimensions of the track reinforcement system can also be influenced by the [17]:

- changes to the sleeper substructure and water and soil conditions resulting from the planned track superstructure replacements and elevation adjustments,
- additional reinforcement in the form of geogrids, soil stabilisation, etc,
- anticipated reduction in the thickness of the necessary reinforcement, resulting from the drying up and increased bearing capacity of the soil after the execution of the drainage system,
- economically permissible excess thicknesses of protective layers,
- lengths of transition zones between sections with different substructure,
- unification of the construction of reinforcements in both tracks, e.g., possibility to use the same materials in all sections of the railway track,
- local conditions (e.g., restrictions on the thickness of the reinforcement) and the intended technology of the works.

4.2 Example of track reinforcement technology

The example concerns track reinforcement with a system of protective layers, built up using the AHM 800-R [17]. The procedure is outlined below.

- **Selection of protective layer material**

If the condition of the subgrade is unsatisfactory, a protective layer of type material, e.g., unsorted stone 0/31.5 mm, should be adopted. The necessary parameters (criteria) for the calculation will be:

- aggregate grain size (according to the grain-size curve),
- aggregate modulus of deformation,
- no mixing of the aggregate of the reinforcement layer with the track ballast,
- resistance to freezing and vibration,
- ensuring the correct interaction of the aggregate with the reinforcing geogrid.

Geotextile materials are selected for individual track sections using the principles:
- the thickness of protective layers is assumed in the range of 0.15÷0.40 m;
- if necessary, the thickness of excessively thick layers is reduced by using reinforcing geogrids,
- thicknesses of designed layers are rounded off to multiples of 0.05 m.

- **Surface upgrading process**

Removal of the upper part of the sleeper base, taking into account the height of the existing and new superstructure and the planned height corrections to the track, followed by the laying of the new superstructure. The purpose of this treatment is to bring the track bed to the condition after the superstructure replacement.

The final work should consist in checking the filtration condition at the interfaces of the planned protective layers with the substructure and, if necessary, applying a separating or filter-separating fleece.

- **Methods for strength calculation**

There are two groups of methods for calculating the protective layer [11, 17]:
- based on the modulus of deformation (the conditions of substructure operation are not taken into account),
- based on tension (a more precise method, as the following factors are taken into account: actual track loads, superstructure, properties of the subsoil, durability of the substructure); the method requiring numerical assistance.

- **Dimensioning of track superstructure reinforcement on the basis of deformation modules** [11, 17, 21]

The method involves determining the thickness of the h_o protective layer of material characterised by the modulus E_o, such that, when laid on the local soil with modulus E_g, the equivalent subgrade modulus E_e is at least equal to the required subgrade modulus E_{e2}.

For example, for the data according to Fig. 7 (E_g = 30 MPa, E_o = 150 MPa) [11, 17, 21] and the required modulus E_{e2} = 80 MPa, it should be assumed that $E_e = E_{e2}$. The task is to calculate the quotients:

$$E_g/E_o = 30/150 = 0{,}200 \tag{19}$$

$$E_e/E_o = 80/150 = 0{,}533$$

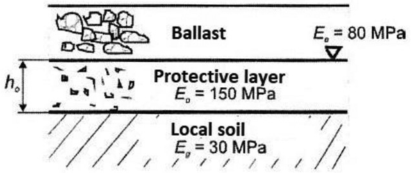

Fig. 7. Data for calculations using the modules of deformation method [11, 17, 21]

For the above quotients, from the DORNII nomogram given in Figure 8 [11, 17], it is read out:

$$h_o/D = 1{,}05 \tag{20}$$

where:

D = 0,30 m is the diameter of the plate of the VSS used for the load tests [12, 14].

Therefore, the thickness of the necessary protective layer: h_o = 1,05 D = 1,05 · 0,30 = 0,315 m (the thickness of the h_o layer can also be read on the horizontal scale of the nomogram given in Fig. 8).

In addition to the single reinforcing layers, double-layer track bed coverings are also designed. The procedure is then as follows [11, 21]:

- for the lower part of the covering with thickness h_o and modulus E_o the quotients E_g/E_o and E_e/E_o should be calculated
- then from the nomogram (Fig. 8) the E_e/E_o is read off,
- on this basis, the E_e, modulus is determined, which is the equivalent modulus for a substructure reinforced with a single layer (Fig. 7),
- then the calculations are repeated for the upper layer; in that case the previously determined equivalent modulus E_g is taken as the base modulus of this layer E_e,
- the achieved equivalent modulus of E_e for the floor of the uppermost layer must not be less than that required for the subsoil E_{e2}.

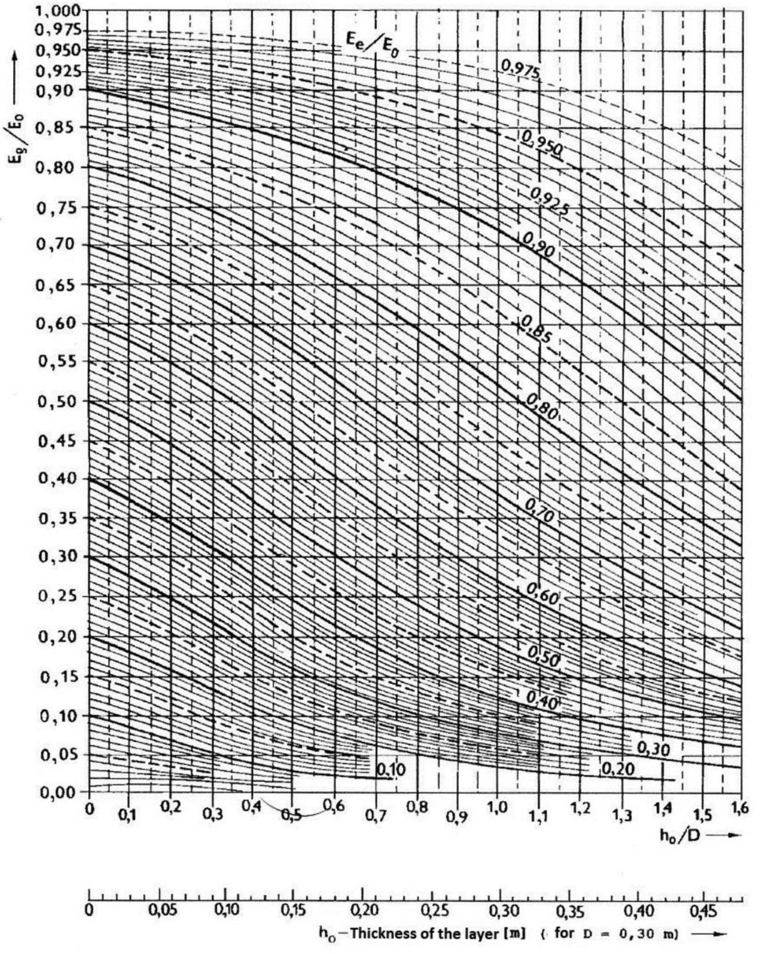

Fig. 8. The DORNII nomogram [11, 17]

The equivalent modulus for a single layer laid on the substrate can also be calculated from the formula [11, 21]:

$$E_e = E_p \{1 - 2/\pi \, [1 - 1 \, (n)^{-3,5}] \arctan [n \, h_w \, (D)^{-1}]\}^{-1} \qquad (21)$$

in which:

E_p – modulus of elasticity of the bedding of the layer,
D = 0,30 m – the diameter of the slab used for the test loads [7, 8],
h_w – layer thickness,
n – parameter expressed by the relation:

$$n = [E_w (E_p)^{-1}]^{0,4} \tag{22}$$

where:

E_w – modulus of elasticity of the layer material.

- **Dimensioning of track reinforcement based on tension**

The tension method consists of checking the tension values in the ceilings of the individual subgrade soil layers. When it is found that the existing tension is greater than the permissible calculated for these soils, the thickness of the designed protective layer should be increased. The calculations are carried out in the following order [17, 24, 25].

1. **Calculation of rail pressures on sleepers** [17, 24, 25]

The vertical axle loads of the rolling stock Q augmented for traffic dynamics are assumed for the calculations. Therefore the following dynamic coefficients shall be taken:

- for the vehicle speed $v \leq 100$ kmph – according to Schramm:

$$k_d = 1 + 0{,}000015\ (3\ v^2 - 0{,}01\ v^3) \tag{23}$$

- for the vehicle speed $v > 100$ kmph – according to Eisenmann:

$$k_d = 1 + 0{,}007\ (v - 60) \tag{24}$$

The forces acting on the foundation of the sleepers are determined assuming that the rails are continuous beams, founded on an elastic foundation. The rail pressure $P_{s\text{-}p}$ on the sleeper is calculated from the formula:

$$P_{s\text{-}p} = 0{,}5\ a\ k_{s,p\text{-}s}\ \Sigma\ G_i\ \eta_i; \quad i = (1 \div n) \tag{25}$$

where:

a – axial spacing of sleepers,

G_i – dynamic load on the rail from one wheel of the vehicle ($G_i = 0{,}5\ Q_i$);

Q_i - vertical axle load of the vehicle;

η_i – ordinates of the influence line of settlement of an infinitely long beam, founded on a continuous elastic foundation;

$k_{s,p\text{-}s}$ – the coefficient of relative stiffness of the ground and rail, calculated according to the relation:

$$k_{s,p\text{-}s} = [U\ (0{,}25\ E_s J)^{-1}]^{0{,}25} \quad [\mathrm{m}^{-1}] \tag{26}$$

in which:

E_s = 2,1 10^6 [kN/m^2] – modulus of elasticity of rail steel,

J – moment of inertia of the rail cross section with respect to the horizontal neutral axis [m^4].

The *U-factor* found in the above formula is called the modulus of the rail substrate and is considered as the value of the force applied to a unit length of the rail, causing unit settlement of the rail substrate. This coefficient is calculated from the formula [21, 24, 25]:

$$U = C\ \alpha\ l\ b\ (2\ a)^{-1} \qquad [\mathrm{kN/m}^2] \tag{27}$$

where:

C – sleeper pad factor (pressure applied to the unit area of the sleeper base causing unit elastic settlement of the sleeper pad);

α – quotient of the average pad settlement and its settlement under the rail; it is assumed to be 0,7÷0,9;

l – half of the sleeper length,

b – width of the sleeper base,

a – pads axial distance.

The sleeper base factor C [kN/m^3], which also takes into account sleeper settlement, can be estimated either from the compressibility of the sleeper, spacer and shim and the settlement of the sleeper base, or from formulas developed by H. Bałuch [25]:

- for track on concrete sleepers $C = 0{,}885\ (0{,}10\ E_e)^{0{,}67}$ 28)

- for track on wooden sleepers $C = 0{,}400\ (0{,}10\ E_e)^{0{,}70}$ (29)

where:

E_e – deformation modulus of the ground track.

2. **Calculation of tensions in the foundation of sleepers** [17, 24, 25]

The normal tensions σ_z, σ_x and shear tensions τ_{zx} at individual points in the sleeper substrate can be calculated, for example, from Florin's formulae for rectangular coordinates [17]:

$$\sigma_z = 3\ p\ z^3\ (2\ \pi\ R^5)^{-1} \tag{30}$$

$$\sigma_x = 3\ p\ (2\ \pi)^{-1}\ \{x^2\ z\ (R^5)^{-1} + 0{,}33\ (1 - 2\upsilon)\ [R\ (R + z)]^{-1} - \\ - (2\ R + z)\ x^2\ [(R + z)^2\ R^3]^{-1} - z\ (R^3)^{-1}\} \tag{31}$$

$$\tau_{zx} = 3\ p\ x\ z^2\ (2\ \pi\ R^5)^{-1} \tag{32}$$

where:

$\upsilon = 0{,}3 \div 0{,}5$ is the Poisson's ratio.

The calculation must take into account the concentrated tensions in soil layers that are more rigid than their base. For this purpose, the points of application of concentrated forces P_i are shifted to the surface of equivalent layers with thicknesses h, according to the following formula and Figure 9 [17]:

$$h = h_i\ [E_w\ (E_p)^{-1}]^{1/n} \tag{33}$$

in which the n factor should be taken in the range of 2,0÷2,5.

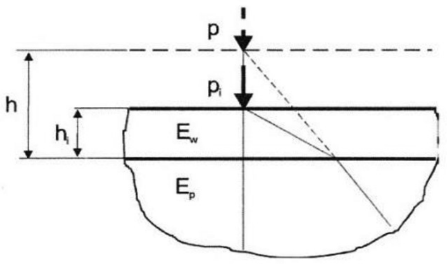

Fig. 9. Odemark's principle of considering the reinforcing layer when calculating tension [17]

When calculating the normal tensions, the geostatic tensions resulting from the self-weight of the soil at the depth under consideration must be added to the vertical tensions σ_z.

3. **Strength check** [24, 25, 26]

Sufficient strength of the subgrade soil at depth z is determined from the condition of:

$$\sigma_z < \sigma_{perm} \tag{34}$$

where:

σ_{perm} – permissible tension of the subgrade soil at depth z, which is calculated as follows:

$$\sigma_{perm} = \sigma_{liminal} (n)^{-1} \tag{35}$$

where:

n – factor of safety (n = 1.5 for repaired substructure; n = 2.0 for upgraded substructure).

In the formula above, $\sigma_{liminal}$ are the limit tensions according to L. Prandtl-A. Caquot, without taking into account the dead weight of the soil below the sleeper (treated as a foundation). The L. Prandtl-A. Caquot formula for calculating the tensions $\sigma_{liminal}$ is of the form [26]:

$$\sigma_{liminal} = \gamma_{ave}\, z\, e^{\pi \operatorname{tg}\varphi} \operatorname{tg}^2 (45^{o} + 0{,}5\, \varphi) + c \cdot \operatorname{ctg} \varphi \; [e^{\pi \operatorname{tg}\varphi} \operatorname{tg}^2 (45^{o} + 0{,}5\, \varphi) - 1] \tag{36}$$

where:

c – soil cohesion at depth z,

φ – angle of internal friction of the soil at depth z,

γ_{ave} – average volumetric weight of the soil to depth z.

The strength of the soil is checked for the floors of all subgrade layers, as sometimes the critical strength is the strength of the soil lying at a considerable depth.

The criterion of sufficient strength can also be checked for the lack of plasticity of the soil (this is the E. Maag-Puzyrewski criterion) [26]:

$$\varphi_{max} < \varphi \tag{37}$$

where:

φ – actual angle of internal friction of the ground,

φ_{max} – the limiting angle of internal friction of the soil, resulting from Coulomb's equilibrium condition; this angle, after taking into account the self-weight of the soil, can be represented as a function of normal tensions.

This relationship has the following form [26]:

$$\sin \varphi_{max} = \{[(\sigma_z - \sigma_x)^2 + 4\, \tau_{zx}^{\;2}]\, [\sigma_z + \sigma_x + 2\, \gamma_o\, (z + c/\gamma \operatorname{tg} \varphi)^2]^{-1}\}^{0,5} \quad 38)$$

4. **Consideration of the durability of reinforcement** [24, 25]

In order to ensure adequate durability of the reinforcement, the calculated permissible tensions are corrected for the operating conditions of the subgrade, i.e., they are multiplied by the coefficient w_k, expressed by the relation:

$$w_k = [k\ T_o\ (k_o\ T)^{-1}]^{0,2} \tag{39}$$

where:

k – the desired maintenance factor for the line under consideration (e.g., for repairs, 1.0; for upgrades, 0.5);
T_o – average gross capacity on all lines (e.g., 10.0 Tg/year);
k_o – average maintenance factor on all lines (e.g., 1.0);
T – predicted gross capacity on the line concerned.

5. NUMERICAL SUPPORT FOR THE DESIGN OF TRACK REINFORCEMENTS

The design of track reinforcements embedded in heterogeneous subgrade (without raising the track) is very labour-intensive, as it requires multiple calculations. In such cases, it is necessary to use appropriate numerical programs to support the design. An example of such a program (developed by E. Skrzyński) is given in Figure 10. The program has the title "Design of track reinforcements (DTR)" [21] and allows, among other things:

- taking into account any number of substrate layers,
- determining the thickness of protective layers,
- generation of track reinforcement designs for individual sections,
- design of multi-layer reinforcements of the track bed (using, among others, geotextiles and stabilized soils),
- determining the parameters of the material of the protective layer that meets all the requirements.

In addition to a knowledge base of various materials, the DTR software includes numerous functions to facilitate design, such as: automatic recalculation of protective layer thicknesses after changing data for the track section under consideration (e.g., after changing pavement characteristics). The DTR software blocks are shown in Figure 10. [21].

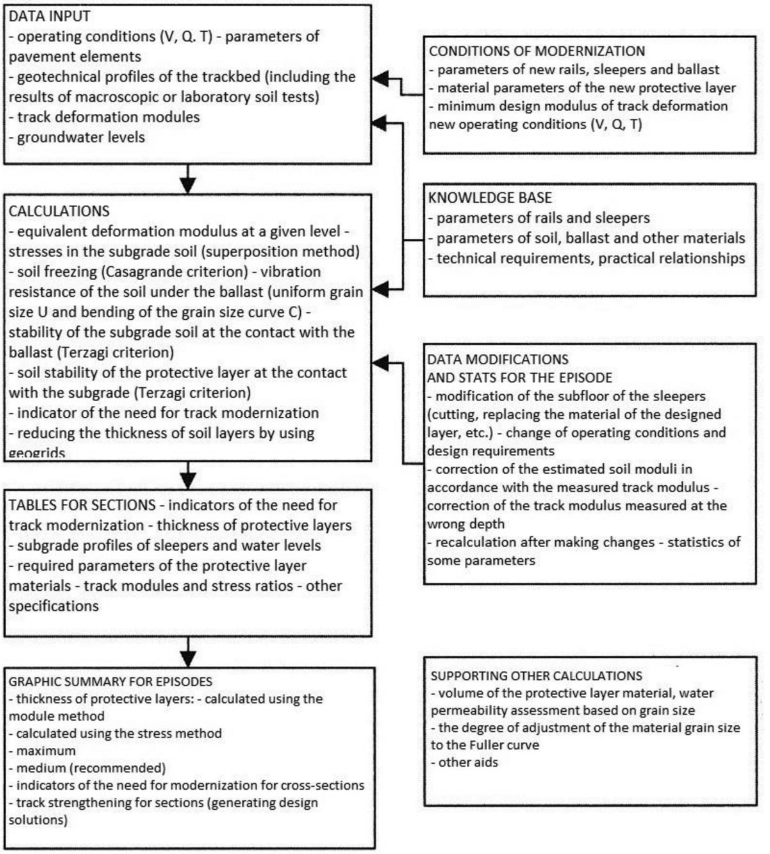

Fig. 10. Software blocks [21]

6. GEO-GRIDS AND TRIAXIAL AS A INNOVATIVE GROUND REINFORCEMENT MATERIAL

Nowadays, plastics, called geosynthetics or geotextiles, are generally used as reinforcing layers for the railroad track bed. In the group of these materials, nonwovens, fabrics, films, meshes (with square or rectangular meshes), geo-mesh, etc., stand out. [3-10, 12-18, 22, 23]. Two-dimensional or three-dimensional reinforcement inserts (so-called mattresses) are created. An alternative to traditional methods of reinforcing the subsoil is the Tensar TriAx geogrid, introduced to the Polish traffic construction market in 2008 [3-10]. The system has become an alternative to traditional methods of soil improvement. The Tensar TriAx structure has received numerous industry recognitions, i.e., the International Geosynthetics Society in

2010 and 2017 – Highways Industry Product of the Year. Tensar TriAx geogrid is a triaxial (hexagonal) material, composed of bars forming equilateral triangles [7]. It is manufactured from a polypropylene sheet in which holes are "punched" and the material is stretched in three directions, so that ribs are obtained that form triangular meshes. Among the most important parameters of the triaxial geogrid, which performs the function of stabilization and reinforcement, is the isotropic stiffness ratio F_{is}. This factor determines the ability to provide similar stiffness values in all test directions [6, 7]. The F_{is} value is estimated from the results of radial stiffness tests, as a quotient of the minimum and maximum stiffness values, which are determined for the corresponding number of measurements taken [2]. Stabilized ballast with a triaxial geogrid obtains improved values of strength parameters, such as bearing capacity and compaction. In addition, the uniformity of vertical displacement, which is a determinant of the uniformity of the structure's performance, is ensured. The principle of operation of the Tensar TriAx geogrid is illustrated in Figure 11. Stabilization of aggregate with this system results in wedging of grains and their inability to move in the horizontal plane. The consequence of these phenomena is the reduction of vertical deformations of the stabilized layer. The installation technology of the Tensar TriAx system is shown in the following figures:

- Fig. 12: unfolded geogrid on geotextile separation and filtration fabric, state before spreading ballast aggregate,
- Fig. 13: placement of aggregate on a layer of triaxial geogrid,

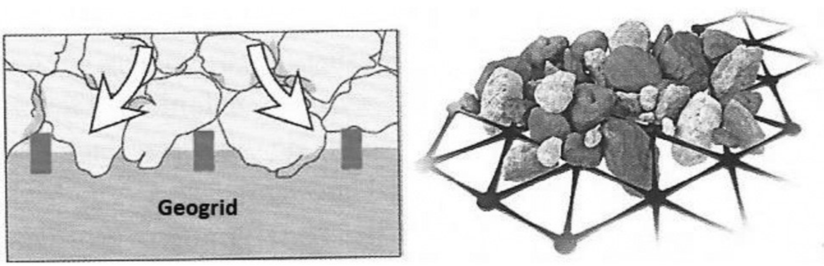

Fig. 11. Principle of functioning of Tensar TriAx geogrid – schemes of blocking and interlocking aggregate grains in the meshes of the geogrid [6]

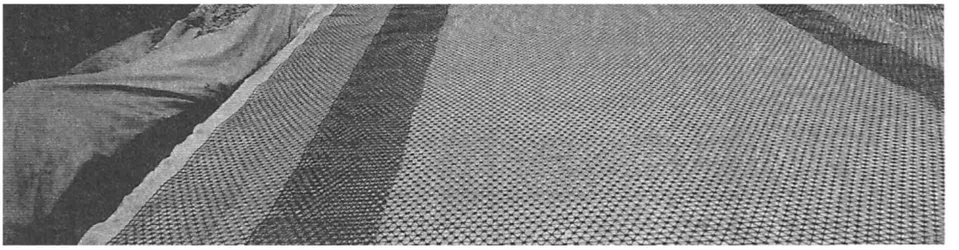

Fig. 12. View of the spread geogrid on the geotextile separation and filtration fabric. Condition before aggregate spreading [6, 7]

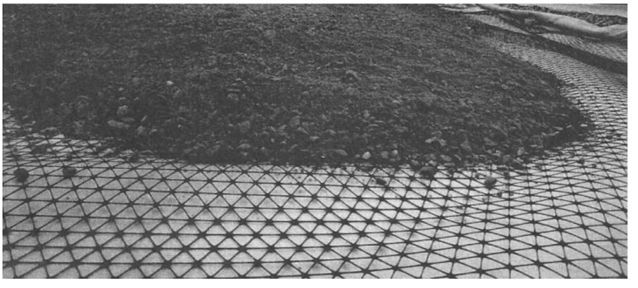

Fig. 13. Aggregate spreading on a triaxial geogrid layer [6, 7]

CONCLUSION

The design of railway road base reinforcement layers requires an analysis of a number of issues, relating to the track surface and base and the reinforcement system. The article discusses some of the issues, i.e.:

- technical requirements for track bed,
- requirements for track bed reinforcement layers,
- outline of strength calculations and dimensioning of the track structure (when designing the reinforcement layers of the railway track bed, it is necessary to know the relevant strength calculations and dimensioning of the track superstructure; these calculations are accompanied by the adoption of a track model)
- calculation methods for the thickness of the reinforcement layer and the decision-making process for selecting the right system,
- numerical support for the design of track reinforcement.

Particularly noteworthy is the information on TriAxial geonets [7, 8, 17], which are an innovative ground reinforcement material.

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Andrzej Surowiecki
The International University
of Logistics and Transport in Wroclaw, Poland
ORCID: 0000-0003-4080-3409
andrzejsurowiecki3@wp.pl

Piotr Saska
The International University
of Logistics and Transport in Wroclaw, Poland
ORCID: 0000-0002-9760-856X
piotrsaska@wp.pl